Journal of Engineering Physics and Thermophysics, Vol. 82, No. 3, 2009

ON THE NON-UNIQUENESS IN THE DEPENDENCE OF THE PARAMETERS OF MASS REMOVAL OF A CARBON MATERIAL ON THE MACH NUMBER IN THE AIR FLOW INCIDENT ON IT

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UDC 621.1.016+536.2

The non-unigueness in the dependence of the parameters of mass removal of a carbon material on the Mach number in the air flow incident on it has been established. This effect occurs in the region of fairly low values of the argument of these functions.

Keywords: *graphite, monotony, continuity, non-unigueness, sublimation, oxidation, equilibrium, heat and mass exchange, heat balance, chemical composition.*

Introduction. Thermal protection by mass removal has long been the basic form of protecting structural elements of space-rocket products and other high-energy devices from high-velocity and high-temperature gas flows affecting them. Carbon-carbon structural materials of high physicomechanical characteristics and a chemical composition virtually no different from the chemical composition of graphite represent the most widespread and promising class of heat-proof materials of such kind.

In [1], it has been shown that allowance for the interference of the processes of kinetic oxidation and sublimation of carbon can produce a substantial change in the dependence of the mass velocities of these processes on the temperature of the material's exterior surface in incident gas flow (below we will use the term "wall" for this surface). In the present work, an attempt is made to intensify and develop these investigations with the aim of studying the features of the dependence of the parameters of mass removal of a carbon material on the conditions in the air flow incident on it. The emphasis is on the case of fairly small velocities of incidence of air where the uniqueness in the dependences of the parameters of material-mass removal on the Mach number is violated, as follows from the results of the performed investigations.

Physicomathematical Formulation of the Problem. We restrict the composition of the gas mixture to the components O, O_2, N, N_2, NO, C, C_2, C_3, CO, CO_2, CN, and Ar and use the assumption of validity of the analogy between the processes of heat and mass exchange in the boundary layer. The value of the heat-exchange coefficient on an impermeable wall will be determined by numerical solution of the equations of a frozen air boundary layer in the vicinity of the critical point of a sphere, whereas the attenuation of heat-exchange intensity due to the injection of the vapor of the material into the boundary layer will be calculated in the linear formulation with an injection coefficient $\gamma = 0.65$.

For a prescribed "wall" temperature T_w, all characteristics of the thermochemical destruction of the material are determined by solution of the system of nonlinear algebraic equations given in [1]. This system of equations involving the Langmuir–Knudsen formula for calculation of the rate of nonequilibrium sublimation of carbon, which is written in the same manner as in [2], the formula for calculation of the rate of kinetic carbon oxidation written in the form of the Arrhenius law, the mass balances of chemical elements on the "wall," and the equations of equilibrium of the chemical reactions of formation of molecular components from the atoms on the "wall" have the form

$$G_\Sigma = G_{ox} + G_{sub}\,, \tag{1}$$

$$G_{ox} = \frac{M_C}{M_O}\,\rho_w\left(C_{O_2w} + C_{Ow} + C_{NOw} + C_{CO_2w}\right)\frac{K_w}{A}\exp\left(-\frac{E_w}{R_{un}T_w}\right), \tag{2}$$

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$$G_{\mathrm{sub}} = \sum_{k=1}^{3} \zeta_k \frac{p_{\mathrm{C}_k\mathrm{w,s}} - p_{\mathrm{C}_k\mathrm{w}}}{A\sqrt{2\pi R_{\mathrm{un}} T_{\mathrm{w}}/(iM_{\mathrm{C}})}}\,, \tag{3}$$

$$(\alpha + G_{\Sigma}) \sum_{i=1}^{n} C_{i\mathrm{w}} \nu_{\mathrm{O}i} = \alpha \sum_{i=1}^{n} C_{ie} \nu_{\mathrm{O}i}\,, \tag{4}$$

$$(\alpha + G_{\Sigma}) \sum_{i=1}^{n} C_{i\mathrm{w}} \nu_{\mathrm{N}i} = \alpha \sum_{i=1}^{n} C_{ie} \nu_{\mathrm{N}i}\,, \tag{5}$$

$$(\alpha + G_{\Sigma}) \sum_{i=1}^{n} C_{i\mathrm{w}} \nu_{\mathrm{C}i} = G_{\Sigma}\,, \tag{6}$$

$$\begin{gathered}
\frac{p_{\mathrm{Ow}}^2}{p_{\mathrm{O_2w}}} = k_{\mathrm{eqO_2}}\,, \quad \frac{p_{\mathrm{Nw}}^2}{p_{\mathrm{N_2w}}} = k_{\mathrm{eqN_2}}\,, \quad \frac{p_{\mathrm{Ow}} p_{\mathrm{Nw}}}{p_{\mathrm{NOw}}} = k_{\mathrm{eqNO}}\,, \quad \frac{p_{\mathrm{Cw}}^2}{p_{\mathrm{C_2w}}} = k_{\mathrm{eqC_2}}\,, \\
\frac{p_{\mathrm{Cw}}^3}{p_{\mathrm{C_3w}}} = k_{\mathrm{eqC_3}}\,, \quad \frac{p_{\mathrm{Ow}} p_{\mathrm{Cw}}}{p_{\mathrm{COw}}} = k_{\mathrm{eqCO}}\,, \quad \frac{p_{\mathrm{Ow}}^2 p_{\mathrm{Cw}}}{p_{\mathrm{CO_2w}}} = k_{\mathrm{eqCO_2}}\,, \quad \frac{p_{\mathrm{Nw}} p_{\mathrm{Cw}}}{p_{\mathrm{CNw}}} = k_{\mathrm{eqCN}}\,.
\end{gathered} \tag{7}$$

In turn the temperature of the "wall" is determined from the condition of fulfillment of the heat balance on it, which is written in the form

$$F(T_{\mathrm{w}}) = A\alpha \left[h_{00} - h_{\Sigma\mathrm{w}} + \sum_{i=1}^{n} h_{i\mathrm{w}} (C_{ie} - C_{i\mathrm{w}}) - G_{\Sigma} (h_{\Sigma\mathrm{w}} - h_{\Sigma\infty}) \right] - \varepsilon_{\mathrm{w}} \sigma T_{\mathrm{w}}^4 = 0\,. \tag{8}$$

It is common knowledge that there are different approaches to calculation of the rate of kinetic carbon oxidation (this issue has been considered in [1] in detail). In the present investigations, we use the simplest of them, within whose framework it is assumed that the components of the boundary layer O, O_2, NO, and CO_2 participate on equal terms in it and that the product of this physicochemical interaction is carbon monoxide (see Eq. (2)).

Results of Investigations. The results of the present investigations are obtained in a wide range of the Mach number of the air flow past the vicinity of the critical point of a sphere of radius 0.1 m at a stagnation pressure of 1 atm.

During the present calculations, we use the same values of the accommodation coefficients as those in [2], i.e., it is assumed that $\zeta_{\mathrm{C}} = 0.2$, $\zeta_{\mathrm{C_2}} = 0.5$, and $\zeta_{\mathrm{C_3}} = 0.1$, $E_{\mathrm{w}} = 1.72 \cdot 10^8$, and $K_{\mathrm{w}} = 4.5 \cdot 10^{10}$ and $4.5 \cdot 10^6$, which, according to the terminology of [1], means the use of "fast" and "slow" kinetics of carbon oxidation (in all the figures, the results of calculations performed with the indicated kinetic constants are shown as solid and dashed curves respectively).

Figure 1 gives some calculated data illustrating the discrepancy F of the heat balance on the "wall" as a function of T_{w}. As is seen, when $K_{\mathrm{w}} = 4.5 \cdot 10^{10}$ the plot of the function in question can arbitrarily be broken into three parts:

(1) the "wall" portion in the region of fairly low temperatures that is characterized by negative values of the derivative $\partial F/\partial T_{\mathrm{w}}$;

(2) the "wall" portion in the region of moderate "wall" temperatures that is characterized by positive values of the derivative $\partial F/\partial T_{\mathrm{w}}$;

(3) the "wall" portion in the region of fairly high temperatures that is characterized by negative values of the derivative $\partial F/\partial T_{\mathrm{w}}$.

The reason for the indicated regularity of variation in $F(T_{\mathrm{w}})$ is as follows. Both the radiation component of the heat balance (8) and the rate of destruction of the material unambiguously increase with this function's argument. For the overwhelming majority of materials of ablation thermal protection whose mass removal is accompanied by the

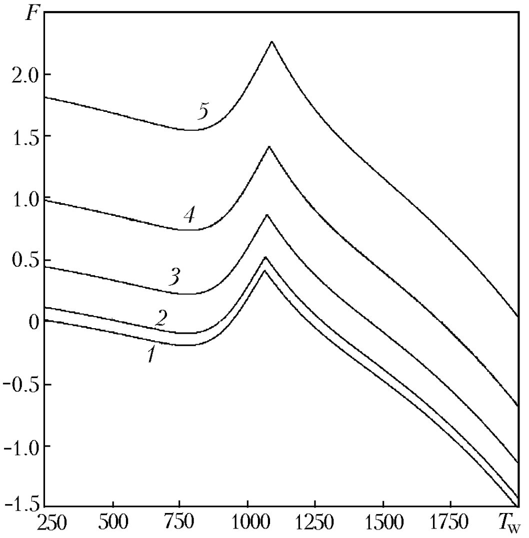

Fig. 1. Discrepancy of the heat balance on the "wall" vs. its temperature: 1) M = 3, 2) 4, 3) 5, 4) 8, and 5) 10. F, MW/m^2; T_w, K.

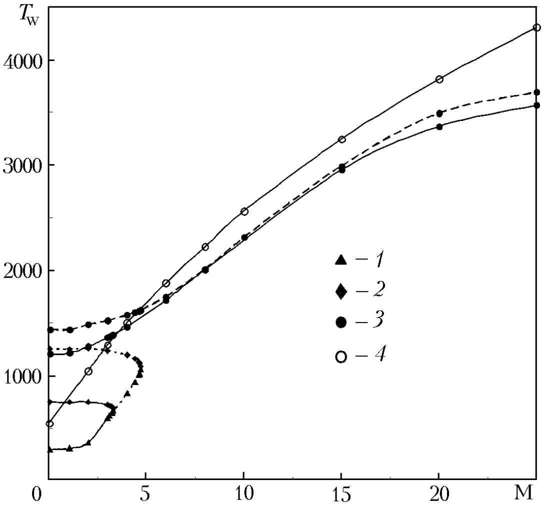

Fig. 2. Temperatures of the "wall," which satisfy the heat balance on it, vs. Mach number: 1) T_{w1}; 2) T_{w2}; 3) T_{w3}; 4) "heat-insulated wall." T_w, K.

absorption of heat, the intensity of convective heat exchange on the "wall," which is described by the first two terms in the heat balance (8), decreases (due partly to the decrease in the heat-exchange coefficient from the effect of injection of the material's vapor into the boundary layer), whereas the difference $h_{\Sigma w} - h_{\Sigma\infty}$, conversely, grows. Thus, for materials of the indicated type, the values of the positive terms in the heat balance (8) decrease as T_w grows, whereas the values of the negative terms grow, which predetermines the monotony of the function $F(T_w)$.

In the sublimation regime of destruction (i.e., in the region of fairly high temperatures) to which, in particular, there corresponds the third portion of the functional dependence in question, carbon materials act as the traditional ablation thermal protection, absorbing a great quantity of heat in the process of mass removal. The form of the function $F(T_w)$ turns out to be traditional here, too.

An analogous situation is observed in the region of fairly low values of the "wall" temperature (to which, in particular, there corresponds the first portion of the dependence in question), when the mass removal of carbon materials is virtually absent.

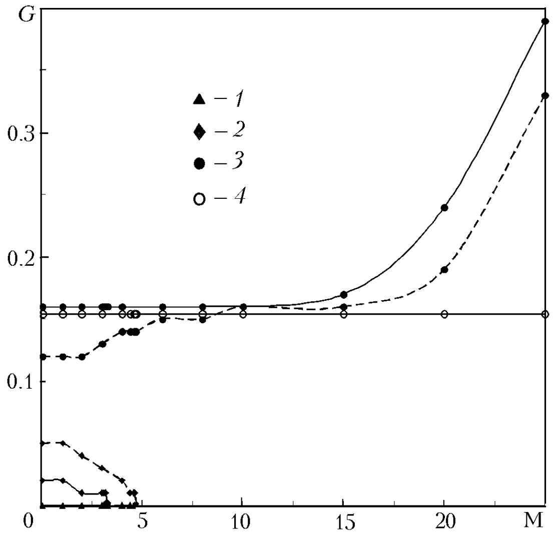

Fig. 3. Total mass velocity of destruction of the material vs. Mach number: 1) $G_{\Sigma 1}$; 2) $G_{\Sigma 2}$; 3) $G_{\Sigma 3}$; 4) diffusion regime.

At the same time, we have an intense oxidation of carbon materials in the temperature range 700–3000 K; the oxidation is accompanied by a considerable heat release. It is precisely this factor that is responsible for the occurrence of the portion with a positive value of the derivative on the plot of the function $F(T_{\text{w}})$.

This character of the functional dependence $F(T_{\text{w}})$ is preserved throughout the considered range of the Mach number in the air flow incident on the material (in both the "fast" and "slow" kinetics of carbon oxidation). However, with decrease in the Mach number and related reduction in the convective heat flux supplied to the material's surface, the values of the function F decrease. The indicated circumstance gives rise to such a situation where, for fairly low Mach numbers, the heat balance on the "wall" (8) can be fulfilled for three T_{w} values for which we will use the notation $T_{\text{w}1}$, $T_{\text{w}2}$, and $T_{\text{w}3}$ in their decreasing order. The dependences $T_{\text{w}l}(\text{M})$ and $G_{\Sigma,l}$ (M) = $G_{\Sigma}[T_{\text{w}l}(\text{M})]$ are given in Figs. 2 and 3. Thus, the minimum temperature value of the indicated set, i.e., $T_{\text{w}3}$, corresponds to the case of an extremely low rate of destruction of the material, because of which this temperature is virtually no different from the so-called "temperature of a heat-insulated wall" (see Fig. 2); the value of the "wall" temperature $T_{\text{w}1}$ corresponds to the regime of a developed process of removal of the material's mass (close to a diffusion or sublimation regime (see Fig. 3)); the value $T_{\text{w}2}$ corresponds to the stage of relatively low-intense kinetic oxidation of carbon, which can arbitrarily be called the regime of "glow combustion." A consequence of the indicated form of the $G_{\Sigma}(\text{M})$ and $T_{\text{w}}(\text{M})$ functions is the dependence of the parameters of mass removal of the carbon material not only on the Mach number, but on the sign of the derivative $d\text{M}/d\tau$ as well.

Indeed, if, under the conditions in question, the Mach number gradually decreases from fairly high values (which corresponds to the braking of a spacecraft), the regime of developed removal of the material's mass, characterized by increased G_{Σ} and T_{w} values, is realized. The dependences $G_{\Sigma}(\text{M})$ and $T_{\text{w}}(\text{M})$ turn out to be monotonic and continuous. Under other flight conditions, disturbance of the monotonicity and continuity of the dependences in question is possible in the regime of braking of a spacecraft, too. Conversely, if the Mach number gradually increases from its fairly low values (which corresponds to the acceleration of a spacecraft), first we have the regime of insignificant removal of the material's mass, which subsequently transforms stepwise to the regime of a developed process. Naturally, the $G_{\Sigma}(\text{M})$ and $T_{\text{w}}(\text{M})$ functions become discontinuous.

CONCLUSIONS

1. It has been established that for fairly small velocities of incidence of the oxidizing gas flow on the carbon material, there is non-uniqueness in the dependence of the characteristics of its mass removal on the Mach number.

2. It has been shown that under the conditions occurring in braking of a spacecraft, the regime of developed removal of the carbon-material mass is realized and the continuity of the dependence of its basic characteristics on the

Mach number is preserved. At the same time, in acceleration of the spacecraft, we have a stepwise transition from the regime of insignificant removal of the carbon-material mass to the regime of its developed mass removal with a discontinuity of the dependence of the basic characteristics of its material on the Mach number.

This work was carried out with financial support from the Russian Foundation for Basic Research (project 06-01-00575).

NOTATION

A, coefficient of convective heat exchange on an impermeable "wall," kg/(m^2·sec); C, mass concentration; E_{w}, activation energy, J/kmole; G, mass velocity in fractions of the coefficient of heat exchange on an impermeable "wall;" F, discrepancy of the heat balance on the "wall," W/m^2; h, enthalpy, J/kg; K_{w}, kinetic constant, m/sec; k_{eq}, equilibrium constant of the chemical reaction of formation of a substance from the atoms, Pa^a (a = 1 and 2); M, molar weight, kg/kmole; M, Mach number in the incident flow; p, pressure, Pa; R, gas constant, J/(kmole·K); T, temperature, K; α, heat-exchange coefficients in fractions of its value on an impermeable "wall;" γ, injection coefficient; ε, emissivity factor; ν, weight content of the chemical element in the substance; ρ, density, kg/m^3; σ, Stefan–Boltzmann constant, W/(m^2·K^4); ζ, accommodation coefficient. Subscripts and superscripts: 00, braking, stagnation; ∞, unheated material; e, external boundary of the boundary layer; i and j, component numbers; k, number of atoms in a purely carbon substance; l, problem's solution number; n, number of components in the gas mixture; ox, oxidation; s, saturated vapor; sub, sublimation; un, universal; w, "wall;" Σ, total; eq, equilibrium.

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